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TOPOLOGICAL SYNTHESIS OF EPICYCLIC GEAR MECHANISMS USING GRAPHICAL TECHNIQUE

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ABSTRACT

In this paper, a new graphical approach for automatic enumeration of epicyclic gear mechanisms is presented using the concept of acyclic graph to enumerate displacement graphs. A new graphical code method has been introduced for automatic evaluation of displacement graphs. This graphical code has been applied as a detection tool for open graph, redundant links and isomorphic graphs. A computer program "GRSY-EGM" has been developed for automatic enumeration of the displacement graphs as will as automatic detection of isomorphic graphs and open graphs and graphs with a redundant link using the concept of graphical code without using the concept of adjacency matrices. This simplified methodology has been successfully applied for the enumeration of epicyclic gear mechanisms with 7 and 8-links having at least 4 coaxial links, two level vertices, at most tree planet gears.

KEYWORDS: "GRSY-EGM", Epicyclic Gear Mechanisms

NOMENCLATURE

N	Number of Links of an Epicyclic Gear Mechanism,
EGMs	Epicyclic Gear Mechanisms,
EGTs	Epicyclic Gear Trains,
f-Circuit	Fundamental Circuit,
PI	Matrix Representing First Level Graphical Code,
PΠ	Matrix Representing Second Level Graphical Code

INTRODUCTION AND DEFINITIONS

The generation of all configuration of an epicyclic gear mechanism can be processed in a systematic approach in which the mechanism is separated from its function, and then the kinematic structures of the same type are enumerated with the aid of graph theory. Finally, each kinematic structure obtained is evaluated according to the functional requirements of such a mechanism. A random number technique and linkage characteristics polynomial has been presented by Tsai 1986 [4] as an efficient method for the identification of isomorphic graphs. Rotation graphs of n-link mechanism have been used to enumerate (N+1) link mechanism. The same concept was applied for enumeration process by matrices. Chatterjee and Tsai 1995 [5] defined the canonical graph of epicyclic gear mechanisms (EGMs) to formulate a systematic methodology for the enumeration of EGMs with up to ten-links using the canonical graph representation. This methodology becomes more complex as the number of links grows. But, it is considered a direct enumeration process. Hsu 1999 [9] presented the acyclic graph as an efficient graph representation for the enumeration process of automatic transmission epicyclic gear mechanisms. The displacement graph was defined with respect to the acyclic graph and the

corresponding adjacency matrices have been derived. The structural code method, derived from the adjacency matrices, has been utilized for the identification of structural isomorphism. An atlas has been constructed using this methodology for epicyclic gear mechanism with up to 10-links having at least four coaxial links and two level vertices. Some additional constraints have been considered by Hsu et al. 2001 [10] for the enumeration of multi speed automotive automatic transmissions. Such that, the graphs are rotationally non-isomorphic, containing at most three planet gears. As a result, an atlas of epicyclic gear mechanism with up to 10-links having four, five and six coaxial links has been presented. In the following sections the fundamental rules of epicyclic gear mechanism and displacement graph representation will be introduced. Then the proposed methodology of enumeration and evaluation will be presented.

Epicyclic Gear Mechanisms

An epicyclic gear mechanism typically consists of a one degree of freedom epicyclic gear train supported by the casing on one axis. The coaxial links are mounted on concentric bearing that are housed in the casing. It is shown that, in most applications, only coaxial links of an epicyclic gear mechanism are used as input, output or fixed. Otherwise one of the links will have its axis moving in space and it will not be possible to connect it to any type of other reduction units or other systems. In the most of epicyclic gear mechanisms the output link is never changed. The desired reduction ratios can be obtained by changing the input and the fixed links. The casing of an epicyclic gear mechanism is a unique link in its kinematic structure. Epicyclical gear trains are used in automatic transmission and the differential units of automotives, pulley blocks, wrist watches, gear boxes, robotics etc. Epicyclic gear trains involved in this study should conform to the following rules [4, 10]:

R1. *The mechanism should obey the general degrees of freedom equation, i.e. no special proportions are required to ensure the mobility of an epicyclic gear train.*

R2. *The mechanism should be planar and its joints are binary.*

R3. *Mechanisms with partial mobility or with partially locked structure should be excluded. In other words the mechanism contains no embedded structures.*

R4. *Each gear should have a turning pair on its axis, and each link in a gear train must have at least one turning pair in order to maintain constant center distance between each gear pair.*

R5. *The mechanism of m-speeds must contain at least m-coaxial links; such that the output speed can be got from one of the coaxial links.*

R6. *Each coaxial link that represents a sun or a ring must has at least one geared edge with another link that represents a planet ; unless the mechanism can not be operated.*

R7. *The mechanism does not contain any redundant links*.

R8. *The mechanism should contain at most three planets; where some epicyclic gear mechanisms having more than three planets are not practical for five and six speeds [10]*.

DISPLACEMENT GRAPH

In an acyclic graph representation, the vertex representing the casing will be labeled as “zero vertex” or ground level. Links of an EGM are denoted as vertices, simple joints are denoted as edges and multiple joints are denoted as solid polygons. An edge connection, as a solid edge, represents a revolute joint where an edge connection, as a dashed or colored edges, represents a gear pair. Vertices that are connected by a common solid polygon represents coaxial link connected by

coaxial revolute joints in the mechanism. The vertices connected to the ground-level vertex by one revolute polygon are the first-level vertices. The vertices connected to the ground-level vertex by two solid polygons or edges are the second-level vertices. The joints connecting the coaxial links can be rearranged without affecting the functionality of the mechanism. Figure 2 (a) represents the corresponding displacement graph of mechanism in Figure. 1. General characteristics of a displacement graph [9] can be summarized as follows:

F1: The graph is a two-level graph, where the first-level vertices are connected to the ground-level vertex by one solid polygon. The second-level vertices are connected to the first-level vertices by one solid polygon or edge.

F2: The graph , representing N-link mechanism, contains N vertices ,N-3 geared edges, and one pendant vertex as the ground-level vertex.

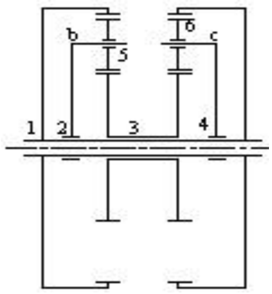

Figure 1: Epecyclic Gear Mechanism

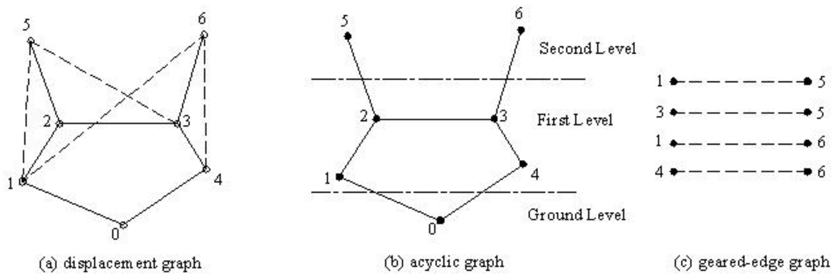

Figure 2: Displacement Graph Representation

A fundamental circuit (f-circuit) is a circuit consists of three vertices, one geared edge and two revolute polygons or edges. The vertex, which is not incident to the geared edge, is called a transfer vertex. The displacement graph of N-vertices EGM is a combination of N-vertices acyclic graph and N-3 geared edges as shown in Figure. 2. In such a graph, there are seen links and four geared joints.

F3: *The sub-graph obtained by removing all the geared edges from the graph is an N-vertex, two level acyclic graphs.*

Isomorphism

Two graphs are said to be isomorphic if there exists a one-to-one correspondence between their vertices and edges which preserves the incidence and the labeling. Isomorphism is considered a problem in the enumeration process such that it represents identical graphs or mechanisms. So, isomorphic graphs should be identified to specify real atlas for graphs under consideration.

Rotational Isomorphism

It is shown that displacement graphs having coaxial second-level vertices are rotationally isomorphic to one displacement graph without any coaxial second-level vertices [10]. For example planets 5, 6 in Figure 3(a) are coaxial but

they are not coaxial in Figure3 (d). Hence, the corresponding rotation graphs are identical as shown in Figure. 3 (c, f). This means that an acyclic graph having coaxial second-level vertices is rotationally isomorphic to one acyclic graph without any coaxial second-level vertices, see Figure. 3 (b,e). This characteristic is considered the first test of isomorphism in enumeration process such that it reduces the number of acyclic graphs that are going to be enumerated.

F4: *The graphs of different EGMs are rotationally non-isomorphic.*

F5: Adding a geared edge to the acyclic graph forms a f-circuit contains one transfer vertex.

The displacement graph should has the following rules:

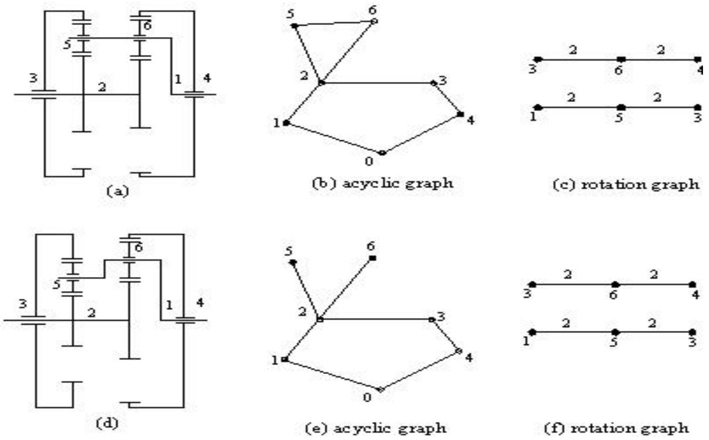

Figure 3: Rotational Isomorphism in Displacement Graph

F6: *The graph has a t least four first-level vertices and at most three second-level vertices*.

The displacement graph must obey the fundamental rules R1-R8 of the epicyclic gear mechanism

F7: *The first level vertices are connected to the root by only thin edges.*

F8: *No geared edges can be incident to the root.*

GRAPH ENUMERATION

Using notation of acyclic graph for enumeration process is more efficient such that it provides a direct enumeration process; it is easier to be graphically computerized and simple as the number of links grows. A unique arrangement of vertices for each graph facilitates an accurate detection of isomorphic graphs. Since the displacement graph is a combination of acyclic graph and a corresponding geared edge graph, then the enumeration process can be divided into two stages. The first is to enumerate the admissible acyclic graphs of n-link mechanism. The second is to add gear edges for each acyclic in all possible arrangements to have all displacement graphs.

Enumeration of Acyclic Graph

The first step in the enumeration process is to have the admissible acyclic graphs for the given n-link mechanism. These graphs are called initial acyclic graphs. That can be described in three steps.

Arrangement of Vertices in the Two Levels

For n-link mechanism, let K refers to the number of first level vertices. So there are (*K+1*) vertices in the ground and the first level. Thus we have the remaining (*N-K-1*) vertices in the second level. For example 8-limk mechanism is represented by 8-vertices arranged as shown in table 1.

Table 1: Arrangement of Vertices at Levels for 8-Link Mechanism

		Ground Level	First Level	Second Level
8-link	Arrangement 1	1	4	3
	Arrangement 2	1	5	2
	Arrangement 3	1	6	1

Enumerate Initial Acyclic Graphs

Each arrangement in Table 1 may generates more than one acyclic graph depending on the nature of the revolute joints in the connection between the first and the second level vertices and that if there are second level coaxial vertices or not.

The second level vertices are connected to the first level vertices by revolute edge or polygon in all possible ways to enumerate all admissible acyclic graphs as illustrated in Figure. 4 for 8-link mechanism. Each admissible acyclic graph has a set of identification numbers a-b-xx where 'a' is the number of links of the given mechanism (N), 'b' is the number of first-level vertices (K), and 'xx' is an order number

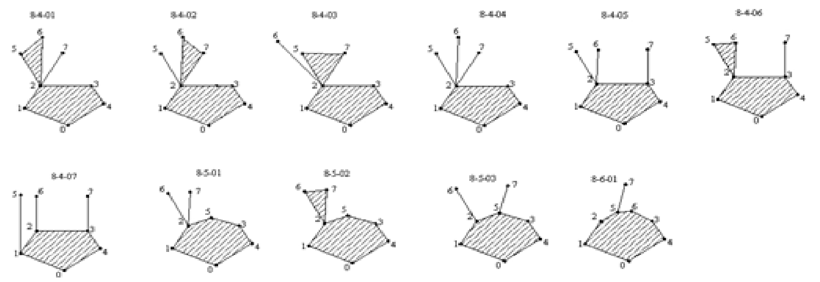

Figure 4: Admissible Acyclic Graphs for 8-Link Mechanism

Rotational Isomorphism

According to the fundamental rules of an acyclic graph, F4, there are some graphs having second level coaxial links which are rotationally isomorphic with those having no second level coaxial links. Applying this fact to 8-link initial acyclic graphs for example, so graph 8-4-01, 8-4-02, and 8-4-03 are rotationally isomorphic with graph 8-4-04. Therefore, these graphs should be excluded from the enumeration procedures. The rotationally non-isomorphic acyclic graphs for eight-link mechanisms have been enumerated in Figure. 5.

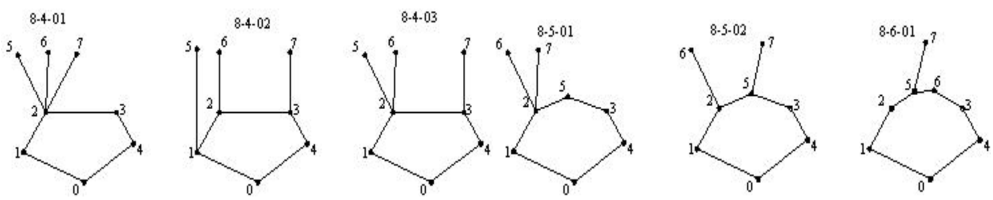

Figure 5: Rotationally Non-Isomorphic Acyclic Graphs for 8-Link Mechanism

Enumeration of Displacement Graphs

The following methodology is applied to generate all displacement graphs using a graphical approach. Each step has been involved in the developed computer program as a subprogram. The acyclic graph 8-5-01 will be used as an illustration example.

Enumeration of All Admissible f-Circuits

Each acyclic graph has m-admissible fundamental circuits. Such that they obey the fundamental rules of a displacement graph F2. The acyclic graph 8-5-01 has nine admissible f-circuits. Table 2 presents the vertices involved in these f-circuits and their associated transfer vertices.

Table 2: Admissible Fundamental Circuits for Graph 8-5-01

f-Circuit No.	Gear Edge Vertices	Transfer Vertex	f-Circuit No.	Gear Edge Vertices	Transfer Vertex
1	6-7	3	6	7-1	3
2	6-1	3	7	7-2	3
3	6-2	3	8	7-4	3
4	6-4	3	9	7-5	3
5	6-5	3			

Identification of the Number of Geared Graphs

Based on the fact that any geared graph can be obtained by selecting any (*N-3*) f-circuits from m f-circuits (F1). Thus, there are C (*m, N-3*) sequences. Each sequence generates one geared graph. For graph 8-5-01, N=8, m=9. So there are c(*9,5*)=*126* sequences can be enumerated.

Enumeration of f-Circuits Sequences

Each geared graph from C(m,N-3) graphs has an arrangement sequence formed by numerated (N-3) f-circuits involved in this displacement graph. For example, the sequence (1,2,3,4 and 9) means that the geared graph is formed by the f-circuits having the ordering numbers 1,2,3,4 and 9 of the admissible f-circuits in table 2. Some of enumerated sequences for graph 8-5-01 are given in Table 3.

Transformation of Sequences into Graphical Arrangements

Therefore, we have all possible sequences of fundamental circuits. Each sequence is applied individually in a graphical form to the desired acyclic graph in order to have one displacement graph. For example the arrangement (1,2,3,4 and 9) is applied graphically in Figure. 6. So all sequences can be transformed to have all displacement graphs for the considered acyclic graph.

Table 3: Arrangement of f-Circuits

Sequence No.	Arrangement of f- Circuits in Each Sequence
1	1, 2, 3, 4, 5
2	1, 2, 3, 4, 6
……….	……………….
5	1, 2, 3, 4, 9
………..	………………….
105	2, 6, 7, 8, 9
…………	…………………
125	4, 6, 7, 8, 9
126	5, 6, 7, 8, 9

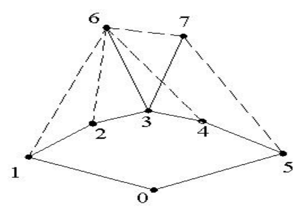

Figure 6: Displacement Graph for Sequence no. 5 of Graph 8-5-01

STRUCTURAL EVALUATION

Enumeration process generates some synthesized geared graphs in which there are some graphs do not match the fundamental rules of the displacement graph. Some of them are structurally isomorphic, some are open graphs, and some

contain redundant links or embedded structures. These graphs should be detected and excluded from the enumeration results. A graphical code is derived for each graph to facilitate evaluation process of graphs under consideration based on an effective priority technique. The final result is a set of non-isomorphic displacement graphs that represents some epicyclic gear mechanisms that can be operated properly.

Priority Technique

There are some rules that have been developed and should be considered before deriving the graphical code of such a displacement graph vertices.

- *Each level vertices are studied individually, noting that if we need to arrange vertices, we will begin with the lower level vertices.*
- *Vertices at the same level having more gear edge-connections should have higher priority.*
- *Vertices at the same level having the same number of geared-edge connections and some revolute joints with another level vertices should be ordered according the number of revolute joints they have ,i.e., the vertex having more revolute joints should has the higher priority.*
- *Vertices at the first level having the same number of geared edge connections and the same number of revolute joints should have the same priority.*
- *Vertices at the first level having no gear edge-connections are numbered according to the number of revolute joint they have. Vertices having more revolute joints should have the higher priority.*
- *Vertices at the first level having no gear edge connections and the same number of revolute joints should have the same priority.*
- *If two vertices at the second level have the same priority applying the previous rules, one of them should has higher priority than another. This can be achieved by considering the number and the type of other joints incident from each vertex at the first level which the considered vertex at the second level is connected to by revolute or geared edge connections. The more geared edge connections or revolute joints are incident from the first level vertex, the higher priority is the associated second level vertex.*

Graphical Code Method

Based on the above rules of priority technique, priority degrees have been derived for each vertex in both levels. For the first level, there is one type of priority degrees depending on the number of geared and revolute joints incident from each first level vertex. For the second level, there is a secondary type of priority degrees beside a primary type. The secondary type depends on other joints incident from each first level vertex that has gear or revolute joints with the associated second level vertex. Graphical code method simplifies the isomorphism process and can be easily computerized. The rules of having the graphical code can be summarized in the next issues:

For First Level Vertices

Each vertex 'x_i' will have a variable $P(x_i)$. The total value of this variable is the sum of the following:

- The number of the geared edge connections (G) incident from vertex ' x ' to the second level vertices multiplied by ' 8 '.
- The number of the revolute joints (R) incident from vertex ' x ' to the second level vertices multiplied by ' 2 '.

$$P(x_i) = 8G + 2R$$

For Second Level Vertices (Primary Priority Degrees)

Each vertex 'y_j' has a variable $P1(y_j)$. the total value of this variable is the sum of the following :

- The number of geared edge connections (G) incident from vertex 'y' to the first level vertices multiplied by ' 8 '.
- The number of the revolute joints (R) incident from vertex ' y ' to the first level vertices multiplied by ' 2 '.

$$P1(y_j) = 8G + 2R$$

For Second Level Vertices(Secondary Priority Degrees)

- The number of other geared edge connections (g) incident from the vertex ' x ' at the first level which the considered vertex ' y_j ' is connected to by gear or revolute joints multiplied by ' 4 '.
- *The number of other revolute joints (r) incident from the vertex ' x ' at the first level which the considered vertex ' y_j ' is connected to by geared or revolute joints multiplied by ' 1 '.*

$$P2(y_j) = \sum (4g + r)$$

Hence, total priority degree for vertex y_j:

$$P(y_j) = P1(y_j) + P2(y_j)$$

Now we have the values of $P(x_i)$ for the first level vertices and $P(y_j)$ for the second level vertices. The graphical code is expressed as two arrays such that the first level has an $1\times$ K array "PI" where K is number of vertices of the first level.

$$PI = [P(x_1), P(x_2) , P(x_3) , \ldots..]$$

The second level has an J×2 array "PΠ" where J is number of revolute joints incident from the first to the second level. Each row represents one revolute joint so there are at most three rows depending on number of second level vertices such as:

$$P\Pi = \begin{bmatrix} P(x_i) & P(y_j) \\ \ldots. & \ldots. \\ \ldots. & \ldots. \end{bmatrix}$$

Where: y_j is the second level link that is connected to the coaxial links x_i with revolute joints. These second level vertices may have its revolute joints with the same coaxial link xi; this means that the values of the first column will be identical. The rules of the graphical code are applied to the displacement graph shown in Figure 8. Graphical code for first and second level can be calculated. Tables 4 (a, b, c) illustrates these results. Such that in the first level, vertices 1, 2, 4, and 5 have only one geared edge incident from them to the second level. So each has the value of ' 1*8 ' ; i.e. ' 8 '. Vertex 3 has two incident revolute joints. So it has the value of ' 2*2 ‘; i.e. ' 4 '. In the second level, vertex 6 has four geared edges and one revolute joint; i.e. ' 4*8 + 1*2 '. In addition to the values added due to the presence of other joints incident from the vertices that have any joint with link 6. From the table 4 (a, b, c) the arrays representing the graphical code of each level are:

$$PI = [\ 8, 8, 4, 8, 8\ ] \ \& \ P\Pi = \begin{bmatrix} 4 & 40 \\ 4 & 32 \end{bmatrix}$$

Table 4 (a): Graphical Code for First Level of Graph in Figure 6

Vertex	No. of Geared Edges ' G '	8G	No. of Revolute Joint ' R '	2R	Total
P1	1	8	--	--	8
P2	1	8	--	--	8
P3	--	--	2	4	4
P4	1	8	--	--	8
P5	1	8	--	--	8

Table 4 (b): Graphical Code for P6 of Graph in Figure. 6

Primary Priority Degrees		Secondary Priority Degrees						Sum
		Vertex	P1	P2	P3	P4	P7	
No. of gear pairs, G	4	No. of gear pairs , G	0	0	0	0	1	
G * 8	32	G * 4	0	0	0	0	4	
No. of revolute joints, R	1	No. of revolute joints, R	0	0	1	0	1	
R * 2	2	R * 1	0	0	1	0	1	
Sum	**34**	Sum	**0**	**0**	**1**	**0**	**5**	**40**

Table 4 (c): Graphical Code for P7 of Graph in Figure. 6

Primary Priority Degrees		Secondary Priority Degrees				Sum
		Vertex	P3	P5	P6	
No. of gear pairs, G	2	No. of gear pairs , G	0	0	3	
8G	16	4G	0	0	12	
No. of revolute joints, R	1	No. of revolute joints, R	1	0	1	
2R	2	R	1	0	1	
Sum	**18**	Sum	**1**	**0**	**13**	**32**

Figure 7 (a) presents the displacement graph shown in Figure 6 after deriving the graphical code for each vertex. Figure 9 (b) presents the same displacement graph after rearranging its vertices according to the derived graphical code.

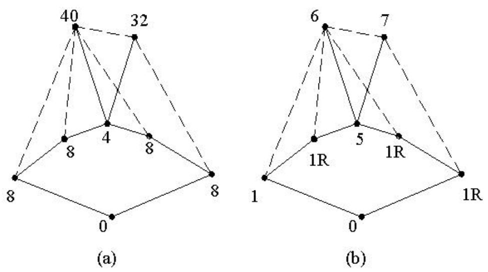

Figure 7: A Displacement Graph of 8-5-01 Acyclic Graph Has 5 f-Circuits

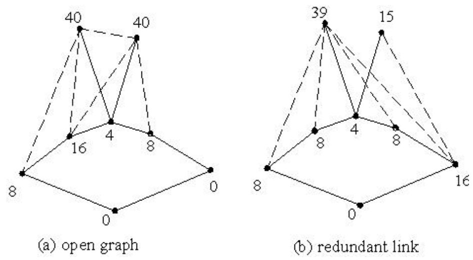

Figure 8: Detection of Open Graph and Redundant Link

Detecting Open Graphs

If there is one or more pendent vertex in the vertex set representing all the fundamental circuits in the associated displacement graph, then this graph would be an open graph. Graphically, if one or more vertex has no geared edges incident from it; then this graph is an open graph. Using graphical code method, if there is one or more ' zero ' element in

both arrays PI; then the graph is an open graph. Applying these conditions to the graph in Figure 7(a) , there is no ' zero ' element in PI. So it is not an open graph. Figure.8 (a) shows another displacement graph for 8-5-01 acyclic graph in which the array representing the first level code is : PI = [8, 16, 4, 8, 0] , that contains a zero element; so this graph is an open graph. This concept is generalized for all displacement graphs to detect all open graphs.

Detecting Redundant Planet Gears

Graphically, the displacement graph has a redundant planet gear if there is a planet at the second level which has only one incident geared edge from the lower level and its carrier. Using the graphical code method if number of connections from second level (first row in table 4-b) equals ' 1 ' then this graph has a redundant planet gear. Applying this to the graph in Figure 7 (a), we find that number of geared connections for vertices 7 and 8 does not equal ' 1 '. So, this graph has no redundant planet gears. However, for the graph in Figure. 8 (b), number of geared connections for vertex 8 equals ' 1 ' . So, this graph has vertex 8 as a redundant planet gear. Applying this concept for all displacement graphs that are resulted from deleting open graphs, we can detect all graphs having redundant planet gears.

Detecting Degenerate Geared Graphs

For each graph of displacement graphs, if any m-link subgraph contains m-1 geared edges and m-1 revolute joints then this graph would be degenerate geared graph or a graph containing embedded structure that should be deleted from the evaluation results. For the graph shown in Figure 7 (a), these conditions are not applicable. So it is not a degenerate geared graph. For the graph shown in Figure.8 (a) the subgraph consists of vertices 2, 3, 6, and 7 contains 4 vertices and 3 geared edges; namely 6-7, 6-2, and 7-2, and 3 revolute joints; namely 6-3, 7-3, and the first level polygon. So the conditions are applicable and then this graph would be a degenerate geared graph. Noting that, in most cases most of the degenerate geared graphs are open graph such that the presence of m-1 geared edges in one subgraph gives a comparatively low chance for a vertex in another subgraph to have geared edges. Therefore, we may notice that the graph in Figure .8 (a) is an example for both open graph and degenerate geared graph.

Isomorphism

Using graphical code method, two graphs are isomorphic if both arrays of the first level PI of two graphs contain the same elements neglecting the arrangement of elements into each array. The second condition is that the two arrays of the second level PΠ of both graphs are identical level neglecting the arrangement of their rows. In other words, if each row of PΠ of one graph has a corresponding one at PΠ of the other graph, then the two graphs are said to be isomorphic. In the previous steps the rejected graphs are detected and deleted. The remaining displacement graphs can represent working mechanisms but there are some isomorphic graphs which need to be deleted. The graphical code method has been efficiently used to detect these graphs. However, for the considered example the remaining displacement graphs can be divided into three groups according to their graphical codes. The first group has a graphical code:

$$\text{PI} = [8, 8, 4, 8, 8] \;\&\; \text{P}\Pi = \begin{bmatrix} 4 & 40 \\ 4 & 32 \end{bmatrix}$$

The second group has a graphical code sets:

$$\text{PI} = [8, 8, 4, 8, 8] \;\&\; \text{P}\Pi = \begin{bmatrix} 4 & 36 \\ 4 & 36 \end{bmatrix}$$

The third group has a graphical code sets:

$$PI = [8, 8, 4, 16, 8] \ \& \ P\Pi = \begin{bmatrix} 4 & 31 \\ 4 & 23 \end{bmatrix}$$

Hence, the final results of the enumeration process for the acyclic graph 8-5-01 are three non-isomorphic displacement graphs shown in Figure. 9.

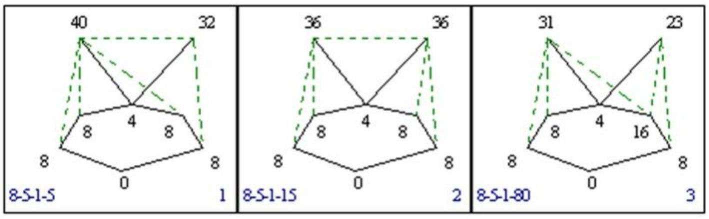

Figure 9: Non-Isomorphic Displacement Graphs for 8-5-01 Acyclic Graph

The same steps with the same sequence should be followed for the remaining acyclic graphs of the 8-link mechanism to have all configurations of 8-link mechanism.

RESULTS AND DISCUSSIONS

A new graphical formulation has been applied for the enumeration of epicyclic gear mechanisms using acyclic graph representation to enumerate all displacement graphs. A new graphical code method has been introduced to derive a code for each displacement graph. This graphical code was successfully applied as a detection tool for open graphs, graphs having redundant links, and isomorphic graphs without using the concept of adjacency matrices. Also, degenerate geared graphs are detected graphically. These rejected graphs are excluded from enumeration results. A computer program "GRSY-EGM" has been developed for automatic enumeration and evaluation procedures. The results of 7 and 8-link mechanisms are in agreement with those of C.H.Hsu [10]. Table 5 summarizes results of 7 and 8 link mechanisms with 4 and 5 coaxial links. These results are illustrated in Figure 10 and Figure. 11.

Table 5: Results for the Enumeration of Displacement Graphs

Acyclic Graph	No. of Enumerated Acyclic Graphs	No. of Nonisomorphic Displacement Graphs
7-4-01	35	2
7-4-02	15	3
8-4-01	792	3
8-4-02	252	4
8-4-03	126	0
8-5-01	126	3
8-5-02	56	4

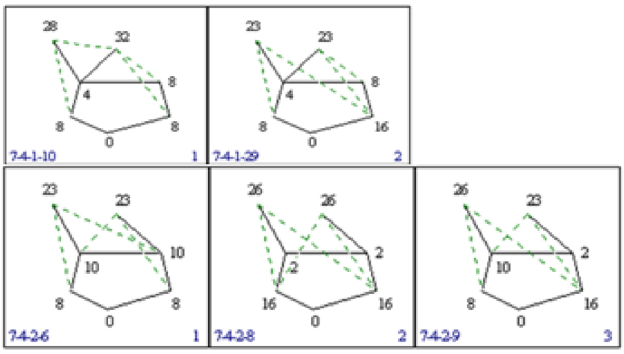

Figure 10: Results of 7-Links Mechanism from "GRSY-EGM"

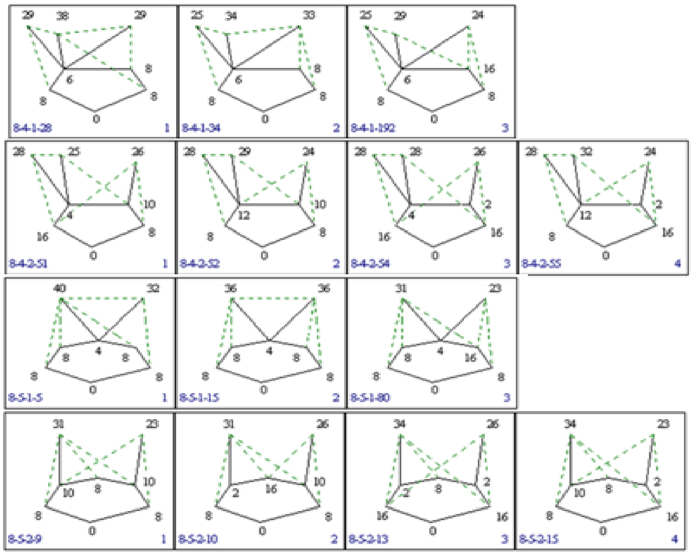

Figure 11: Results of 8-Links Mechanism from "GRSY-EGM"

CONCLUSIONS

In this work, a displacement graph and an acyclic graph and their fundamental rules have been presented. Systematic procedures have been developed and illustrated through various examples to enumerate acyclic graphs and displacement graphs graphically. The priority technique and new graphical code method is correctly applied for identification of open graphs, redundant links and isomorphic graphs. Graphs of epicyclic gear mechanisms of 7 and 8-links have been enumerated using this methodology. A computer program "GRSY-EGM" has been developed for the enumeration process and evaluation process. The proposed methodology can be generalized for mechanisms with more than 8-link.

REFERENCES

1. Freudenstein (1971), *An Application of Boolean Algebra to the Motion of Epicyclic drives* . ASME Journal of Engineerig for Industry, Vol. 93, series B, pp 176-182.

2. Mruthyunjaya, T.S., and Ravisankar, R., 1985, *Computerized Synthesis of the Structure of Geared Kinematic Chains,* Journal of Mechanisms and machine Theory, Vol. 20, pp. 367-387.

3. L. W.Tsai and Lin (1988) , *The Creation of True Two Degree of Freedom Epicyclic Gear Trains*, Number: TR 88-21, Institute for System Research, University of Maryland.

4. L. W.Tsai (1987) *An Application of the Linkage Characteristic Polynomial to the Topological Synthesis of Epicyclic Gear Train*. ASME Journal of Mechanisms, Transmissions, and Automation in Design, 109, No.3, 329-336.

5. Chatterjee, G. and L. W. Tsai (1994) *Enumeration of Epicyclic-type Automatic Transmission Gear Trains*. Transactions of SAE Technical Paper.941012, SAE, Warrendale, PA, U.S.A.

6. L. W.Tsai (1995) *An Application of Graph Theory to the Detection of Fundamental Circuits in Epicyclic Gear Trains*. ASME Journal of Mechanisms, Transmissions, and Automation in Design, Vol. 109, No.3,pp 329-336.

7. Hsieh, H. I. and L. W. Tsai (1996a) *A Methodology for Enumeration of Clutching Sequences Associated with Epicyclic-type Automatic Transmission Mechanisms*. Transactions of SAE Technical Paper 960719, SAE, Warrendale, PA, U.S.A.

8. Hsieh, H. I. and L. W. Tsai (1996b) *Kinematic analysis of epicyclic-type transmission mechanisms using the concept of fundamental geared entities*. ASME Journal of Mechanical Design, Vol. 118, pp. 294-299.

9. Hsu, C. H. (1999) *Systematic Enumeration of Epicyclic Gear Mechanisms for Automotives*. JSME International Journal, Series C, vol. 42, pp. 225-233.

10. Hsu, C. H.and Y.C.YEH and Z.R.YANG (2001) *Epicyclic Gear Mechanisms for Multi-Speed Automotive Automatic Transmissions*. Proc. Natl. Sci. Counc. ROC(A),Vol. 25,No.1, pp. 63-69.